Unit 15

Day 5

RATIONAL ROOTS

8) $x^3+2x^2+9x+18=0$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

	1	2	9	18
1	1	3	12	30
-1	1	1	8	10
2	1	4	17	52
-2	1	0	9	0

$(x+2)(x^2+9)$

$x = \{-2, \pm 3i\}$

Rational Root theorem:

If a polynomial function has integral coefficients, and if it has a rational zero $\frac{p}{q}$, where p and q are relatively prime, then p is a factor of the constant term and q is a factor of the leading term.

Corollary of the rational root theorem:

If the leading coefficient of a polynomial function with integral coefficients is 1 , then any rational zeros of the function are integers and factors of the constant terms.

Factor completely and identify the zeros.

1) $f(x)=4x^3+16x^2+19x+6$

\# of possible zeros: 3

factors of p: $\pm 1, \pm 2, \pm 3, \pm 6$

factors of q: $\pm 1, \pm 2, \pm 4$

possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{1}{4}$

	4	16	19	6
0				6
1	4	20	39	45
-1	4	12	7	$\cancel{3}$ -1
$-\frac{1}{2}$	4	14	12	0

$$\left(x+\frac{1}{2}\right)(4x^2+14x+12)$$

$$\left(x+\frac{1}{2}\right)(2)(2x^2+7x+6)$$

$$(2x+1)(2x^2+7x+6)$$

$$(2x+1)(2x+3)(x+2)$$

$$x=\left\{-\frac{1}{2}, -\frac{3}{2}, -2\right\}$$

Boundedness Theorem:

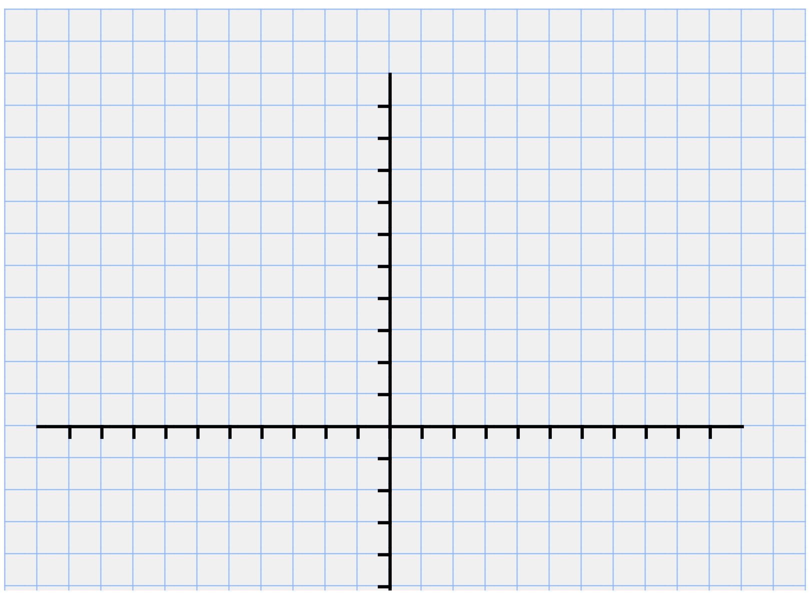
Let f(x) be a polynomial of degree $n \geq 1$ with real coefficients and with a positive leading coefficient. If f(x) is divided synthetically by :

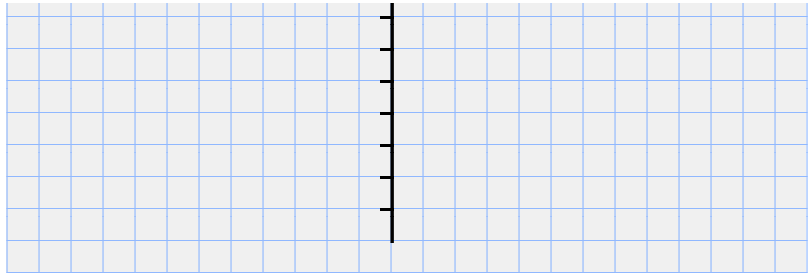
a) $k>0$ and all numbers in the result are non-negative, then f(x) has no zero greater than k.

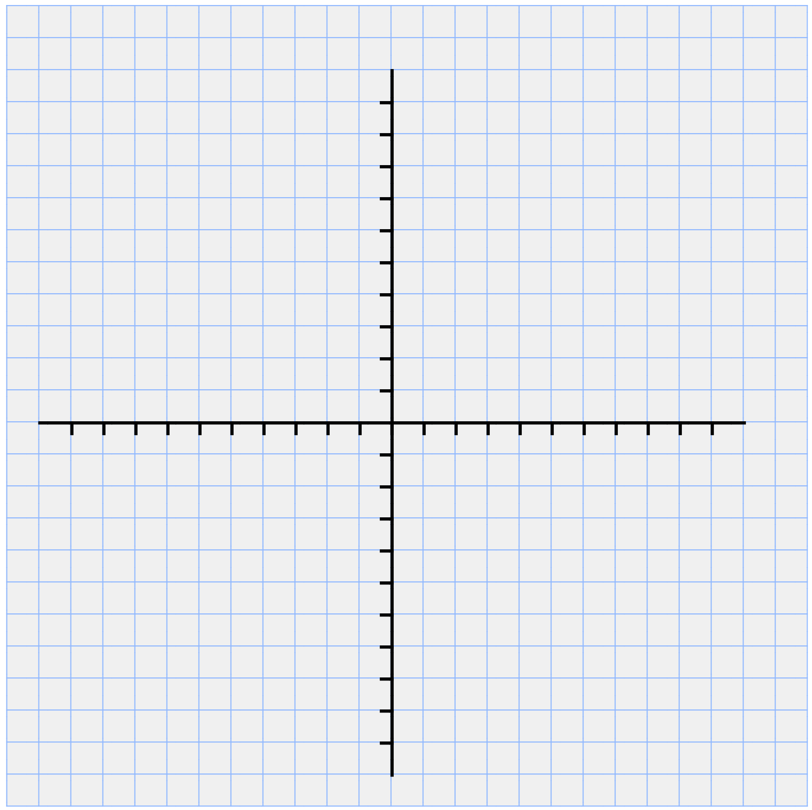
b) $k<0$ and the numbers in the result alternate signs (0 pos. or neg as needed, then f(x) has no zero less than k.

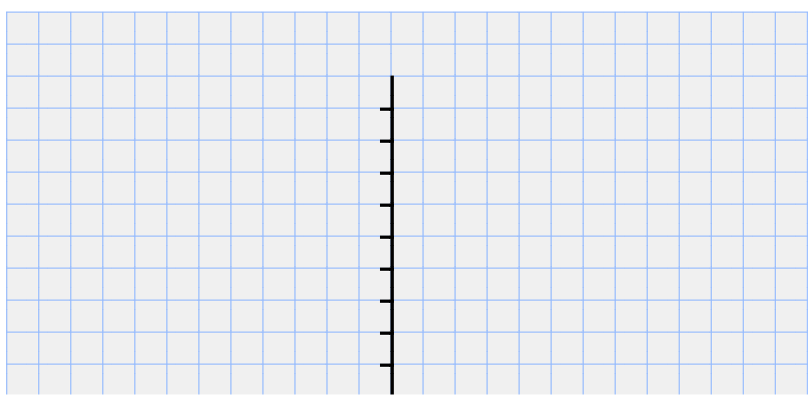
Intermediate value theorem:

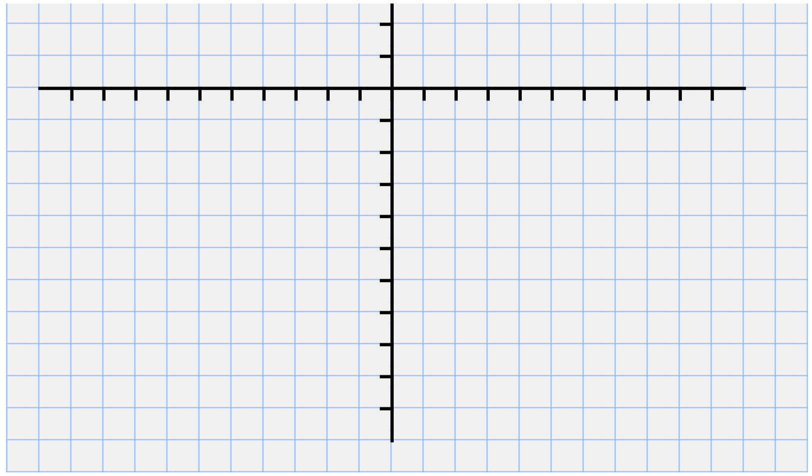
If f(x) is polynomial with only real coefficients , and if for real #'s a and b , the values of f(a) and f(b) are opposite in sign then there exists at least one zero between a and b.











2) $f(x)=2x^4+15x^3+28x^2-9x-36$

\# of possible zeros:

factors of p:

factors of q:

possible rational zeros:

HW pg 300 25-28 all & pg 312-313 33-38, 41-44 all